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SEVERAL NEW OPERATIONAL CALCULUS FORMULAS

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One-dimensional Laplace transforms of some elementary and special functions are given.

The probability of obtaining a precise analytic solution to a given problem by methods of the operational calculus often depends on the presence of appropriate operation formulas in the reference tables [1, 2]. Thus, it is important to publish addenda to these tables. We note that operational methods have recently been applied to the identification of thermophysical properties [3]. The new operation formulas in Table 1 are given in double-column form. The left column contains the function f(x), and the right column, the Laplace transform F(p), where

$$F(p) = \int_0^\infty f(x) \exp(-px)\, dx$$

(Re p > 0, unless otherwise specified). The notation is standard.

TABLE 1

№	$f(x)$	$F(p)$
1	$(1-x)^n$	$\frac{1}{p^n} L_n^{-n-1}(-p)$
2	$\operatorname{ch}\sqrt{x}$	$\frac{1}{p} + \frac{1}{p^{3/2}} e^{1/(4p)} \operatorname{erf}\left(\frac{1}{2\sqrt{p}}\right)$
3	$\cos\sqrt{x}$	$\frac{1}{p} - \frac{1}{p^{3/2}} e^{-1/(4p)} \operatorname{erfi}\left(\frac{1}{2\sqrt{p}}\right)$
4	$\operatorname{ch}\sqrt{x}\cos\sqrt{x}$	$\frac{1}{p} - \frac{\sqrt{\pi}}{p^{3/2}}\left[\sin\frac{1}{2p} C\left(\frac{1}{\sqrt{2p}}\right) - \cos\frac{1}{2p} S\left(\frac{1}{\sqrt{2p}}\right)\right]$
5	$\frac{1}{x}[\operatorname{Ei}(\pm x) - \ln x - C]$	$\operatorname{Li}_2\left(\pm\frac{1}{p}\right)$ Re $p > 1$
6	$\frac{1}{x}[\operatorname{ci}(x) - \ln x - C]$	$\frac{1}{4}\operatorname{Li}_2\left(-\frac{1}{p^2}\right)$ Re $p > 1$
7	$\operatorname{Shi}(x)$	$\frac{1}{p}\operatorname{Arth} p$ Re $p > 1$
8	$\frac{1}{x}[\operatorname{chi}(x) - \ln x - C]$	$\frac{1}{4}\operatorname{Li}_2\left(\frac{1}{p^2}\right)$ Re $p > 1$

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TABLE 1 (continued)

№	$f(x)$	$F(p)$
9	$\frac{1}{\sqrt{x}} \operatorname{erf}(\sqrt{x})$	$\frac{2}{\sqrt{\pi p}} \operatorname{arctg} \frac{1}{\sqrt{p}}$
10	$x \operatorname{erfc}(-\sqrt{x})$	$\frac{1}{\sqrt{p+1}\,(\sqrt{p+1}-1)}$ $\operatorname{Re} p > -1$
11	$\frac{1}{x}[e^x \operatorname{erfc}(\pm\sqrt{x}) - 1]$	$-2\ln\left(1 \pm \frac{1}{\sqrt{p}}\right)$ $\operatorname{Re} p > 1$
12	$\frac{1}{\sqrt{x}} \operatorname{erfi}(\sqrt{x})$	$\frac{1}{\sqrt{\pi p}} \ln \frac{\sqrt{p}+1}{\sqrt{p}-1}$ $\operatorname{Re} p > 1$
13	$\frac{1}{\sqrt{x}} \operatorname{erf}(\sqrt{x}) \operatorname{erfi}(\sqrt{x})$	$\frac{1}{\sqrt{\pi p}} \ln \frac{p+1}{p-1}$ $\operatorname{Re} p > 1$
14	${}_1F_1(\alpha;\ \beta;\ x)$	$(\beta-1)\, p^{\beta-2}\left(1-\frac{1}{p}\right)^{\beta-\alpha-1} \mathrm{B}_{1/p}(\beta-1,\ \alpha-\beta+1)$
15	$\frac{1}{x}[J_0(2\sqrt{x}) - 1]$	$\operatorname{E}i\left(-\frac{1}{p}\right) - \ln\frac{1}{p} - \mathrm{C}$
16	$x^{\nu/2+1}\mathbf{H}_\nu(2\sqrt{x})$	$\frac{1}{p^{\nu+1}} e^{-1/p} \operatorname{erfi}\left(\frac{1}{\sqrt{p}}\right)$
17	$x^{\nu/2+1}\mathbf{L}_\nu(2\sqrt{x})$	$\frac{1}{p^{\nu+1}} e^{1/p} \operatorname{erf}\left(\frac{1}{\sqrt{p}}\right)$
18	$x^{n/2}[J_n(2i\sqrt{x}) - i\mathbf{H}_n(2i\sqrt{x})]$	$p^{-n-1} e^{1/p} \operatorname{erfc}\left(-\frac{1}{\sqrt{p}}\right)$
19	$\operatorname{ber}^2(2x) + \operatorname{bei}^2(2x)$	$\frac{2}{\pi(p^4-64)^{1/4}} \mathrm{K}\left[\frac{1}{2}\left(1-\frac{p^2}{\sqrt{p^4+64}}\right)^{1/2}\right]$
20	$C_{2n+1}^{\nu-n-1}(x)$	$\frac{2(\nu-1)(2-\nu)_n}{n!} e^{i\pi(\nu+n)/2} p^{\nu-n-2} S_{n-\nu+1,n+\nu}(-ip)$
21	$P_{2n+1}(x)$	$\frac{(-1)^n e^{-3\pi i/4}(2n+1)!}{2^{2n}(n!)^2 p^{1/2}} S_{-\frac{1}{2},\,2n+3/2}(-ip)$
22	$P_n(1-2x)$	$\frac{1}{2}\sqrt{\frac{\pi}{p}}\, e^{-p/2}\left[I_{n+1/2}\left(\frac{p}{2}\right) + I_{-n-1/2}\left(\frac{p}{2}\right)\right]$
23	$(1-x)^n P_n\left(\frac{1+x}{1-x}\right)$	$\frac{n!}{p^{n+1}} L_n(-p)$
24	$(1+x)^{n/2} P_n\left(\frac{1}{\sqrt{1+x}}\right)$	$4\left(\frac{1}{4p}\right)^{n/2+1} H_n(\sqrt{p})$
25	$(x^2+1)^{n+1/2} P_{2n+1}\left(\frac{x}{\sqrt{x^2+1}}\right)$	$\frac{(-1)^n(2n+1)!\,2^{-2n}}{(n!)^2 p^{2n+2}} S_{2n+1,\,0}(p)$
26	$(x^2+1)^n P_{2n}\left(\frac{x}{\sqrt{x^2+1}}\right)$	$\frac{(-1)^n(2n)!\,2^{-2n}}{(n!)^2 p^{2n+1}} S_{2n+1,\,0}(p)$
27	$(x^2-1)^n P_{2n}\left(\frac{x}{\sqrt{x^2-1}}\right)$	$\frac{(-1)^n(2n)!\,2^{-2n}}{(n!)^2 p^{2n+1}} S_{2n+1,\,0}(-ip)$
28	$T_n(1+x^2)$	$\sqrt{2}\, O_{2n}(\sqrt{2}\,p)$
29	$T_n(1-x^2)$	$-\sqrt{2}\, i O_{2n}(-\sqrt{2}\, ip)$
30	$xU_n(1+x^2)$	$\frac{2(n+1)}{2n+1} O_{2n+1}(\sqrt{2}\,p)$

TABLE 1 (continued)

№	$f(x)$	$F(p)$
31	$L_n^{\nu}(-x)$	$\frac{(\nu+1)_n}{n!(p+1)}\ {}_2F_1\left(1,\ n+\nu+1;\ \nu+1;\ \frac{1}{p+1}\right)$
32	$x^n L_n^{\nu-n}(x)$	$\frac{(-1)^n}{n!p}(2\sqrt{p})^{-n-\nu}S_{n+\nu+1,\,n-\nu}(2\sqrt{p})$
33	$x^n L_n^{-2n}\left(\frac{1}{x}\right)$	$\frac{2(-1)^n}{n!\sqrt{p}}\ O_{2n}(2\sqrt{p})$
34	$x^n L_n^{-2n-1}\left(\frac{1}{x}\right)$	$\frac{4(-1)^n}{n!(2n+1)}\ O_{2n+1}(2\sqrt{p})$
35	$H_{2n}(x)$	$\frac{(-1)^n(2n)!\,2^{2n}}{p^{2n+1}}\ L_n^{-n-1}\left(-\frac{p^2}{4}\right)$
36	$H_{2n+1}(x)$	$\frac{(-1)^n(2n+1)!\,2^{2n+1}}{p^{2n+2}}\ L_n^{-n-1}\left(-\frac{p^2}{4}\right)$
37	$x^{\mu-1}E_{\rho}(x^{1/\rho}\ \mu)$;	$\frac{p^{1/\rho-\mu}}{p^{1/\rho}-1}$

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